Therefore

$$\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}\,dx = -\tfrac{3}{16}\int_1^{1/2} \frac{1-u^2}{u^2}\,du = \tfrac{3}{16}\int_1^{1/2} (1-u^{-2})\,du$$

$$= \tfrac{3}{16}\left[u + \frac{1}{u}\right]_1^{1/2} = \tfrac{3}{16}\left[\left(\tfrac{1}{2}+2\right) - (1+1)\right] = \tfrac{3}{32}$$

EXAMPLE 7 □ Evaluate $\displaystyle\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$.

SOLUTION We can transform the integrand into a function for which trigonometric substitution is appropriate by first completing the square under the root sign:

$$3 - 2x - x^2 = 3 - (x^2 + 2x) = 3 + 1 - (x^2 + 2x + 1)$$
$$= 4 - (x+1)^2$$

This suggests that we make the substitution $u = x + 1$. Then $du = dx$ and $x = u - 1$, so

$$\int \frac{x}{[\text{illegible}]}\,dx = \int \frac{u-1}{[\text{illegible}]}\,du$$

11. $\int \sqrt{1 - 4x^2}\, dx$

12. $\int x\sqrt{25 + x^2}\, dx$

13. $\int \frac{\sqrt{9x^2 - 4}}{x}\, dx$

14. $\int \frac{du}{u\sqrt{5 - u^2}}$

15. $\int \frac{x^2}{(a^2 - x^2)^{3/2}}\, dx$

16. $\int \frac{dx}{x^2\sqrt{16x^2 - 9}}$

17. $\int \frac{x}{\sqrt{x^2 - 7}}\, dx$

18. $\int \frac{dx}{[(ax)^2 - b^2]^{3/2}}$

equation $x^2 + y^2 = r^2$. Then A is the sum of the area of the triangle POQ and the area of the region PQR in the figure.]